# A Virtual MIMO Path-Time Code for Cognitive Ad Hoc Networks

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***Abstract*—Cognitive radio networks (CRNs) consisting of opportunistic links greatly elevate the networking throughput per bandwidth in future wireless communications. In this letter, we show that by encoding a data packet into several coded packets and transmitting in various time slots, the ene-to-end transmission in the CRN can be equivalently formulated as a physical-layer multiple-input multiple-output (MIMO) communication problem. Two coding schemes are proposed in such a virtual MIMO scenario to enhance the end-to-end communication reliability by exploiting the path diversity. The closed-form expression of the theoretical error rate analysis validates that the proposed path-time codes can significantly improve the error rate performance.**

***Index Terms*—Multiple-input multiple-output (MIMO), cognitive radio network (CRN), ad hoc network, space-time code, path-time code.**

## I. INTRODUCTION

SENSING the unoccupied spectrum and accessing the spectrum opportunities, the cognitive radio (CR) is considered as a key technology to enhance the spectrum efficiency of the future wireless communications. However, the reliable packet transportation over cognitive radio ad-hoc network remains a technology challenge. Specifically, let us first define the *link* as the transmission between two successive nodes, and the *path* as the transmission route between the source and the destination nodes, i.e., the end-to-end transmission. When a packet is transmitted from a CR through the opportunistic relay path (due to the opportunistic transmission), the relay nodes along the path forward the packets to their next available relay nodes. To avoid interfering with the primary system or other CRs, a relay node must sense the link availability before forwarding. If the link is occupied by either the primary user or other CRs, the forwarded signal is queued until the link is available. Consequently, the signals arriving from various relay paths experience different delays and channel gains, and some of them might exceed the time-out period. As the relay paths are opportunistic and likely uni-directional [1], these expired signals are simply discarded, thereby significantly deteriorating the cognitive radio network (CRN) performance.

To combat with this error control problem, Ao and Chen have proposed the end-to-end hybrid automatic repeat request (HARQ) [2] that exploits the code diversity. In this letter, the problem is treated from a different perspective. We map the source node and the destination node to a pair of transmitter and receiver in the physical layer. By transmitting multiple coded packets through multiple relay paths, the end-to-end transmission can then be formulated as a physical-layer multiple-input multiple-output (MIMO) system with the feature that some channels disappear with a certain probability. Using this formulation, the well-developed MIMO techniques in the physical-layer research can be referred and redesigned to improve the transmission quality in the end-to-end CRN transmission.

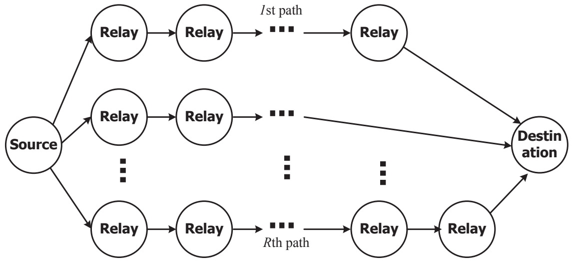

Fig. 1. The topology of end-to-end transmission in CRN.

Based on the permutation matrix and the maximum distance criterion in the coding theory [3], we propose two coding schemes in such *virtual MIMO* scenario that leverage the path diversity in CRN. Note that the distributed space-time code [4] also adopts the physical-layer MIMO techniques to the network layer. However, our scheme takes the CRN opportunistic relay paths into consideration, which makes the system design more challenging. Both the theoretical analysis and the numerical simulations demonstrate the effectiveness of the proposed schemes. Shortly summarized, the contributions of this letter are as follow

- Mapping of the end-to-end CRN transmission to an equivalent physical-layer MIMO system,
- Two proposed coding matrices that significantly improve the error rate performance of the end-to-end CRN transmission, which can be named as *path-time codes (PTCs)*, and
- Closed-form analysis of the error floors for the end-to-end CRN transmission.

## II. SYSTEM MODEL

Fig. 1 illustrates the end-to-end transmission in a multi-hop and multi-path CRN. Suppose that for a pair of source and destination nodes, there exist $R$ link-disjoint paths [5], each consisting of $N_r - 1$ relay nodes, $r = 1, \ldots, R$. The $r$th path gain $g_r(t)$ at time instant $t$ can then be computed by multiplying the link gains of the $N_r$ links..

To model the effect that relay paths are occupied by the primary user or other CRs, we use the two-state discrete-time Markov chain model [6]. Specifically, for the $r$th relay path,

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$p_{01,r}$ and $p_{10,r}$ are respectively defined as the state transition probabilities from the state 0 to 1 and from the state 1 to 0. Such path-level state transition probabilities reflect the link availability and the number of relay nodes in a path. Specifically, the less the link availability or the larger number of relay nodes, the smaller $p_{01,r}$ will be. The availability of the $r$th path can be represented by a Bernoulli random variable $v_r \in \{0, 1\}$, where $v_r = 0$ means that the $r$th relay path is occupied for the entire end-to-end transmission, while $v_r = 1$ indicates that the $r$th relay path is available at least part of the end-to-end transmission duration in which the complete coded packets can be transmitted. Such channel model, though relatively simple, well describes the occupancy/availability of the opportunistic channels and is commonly used [7], [8]. The probability of $v_r$ is given by $P(v_r = 0) = \big(p_{10,r}/(p_{01,r} + p_{10,r})\big)^W$ and $P(v_r = 1) = 1 - P(v_r = 0)$. $W$ is the ratio of the time-out period to a pre-determined period for the entire end-to-end transmission. The larger value of $W$, the longer time-out period the destination node can allow, resulting in smaller $P(v_r = 0)$.

Hereafter, we propose a coded end-to-end transmission, where the source node encodes a data symbol packet $\mathbf{x}$ by a coding matrix $\mathbf{C}(t)$ and then transmits the coded packet $\mathbf{s}(t) = [s_1(t) \ \ldots \ s_R(t)]^T$, both at time instant $t$. The received signal at the destination node can then be represented as

$$y(t) = \mathbf{h}(t)^T\mathbf{s}(t) + n(t) = \mathbf{h}(t)^T\mathbf{C}(t)\mathbf{x} + n(t), \qquad (1)$$

where $\mathbf{h}(t) = [v_1g_1(t) \ \ldots \ v_Rg_R(t)]^T$; $n(t)$ is the additive white Gaussian noise aggregating from all links. In order to enhance the transmission reliability, a single data packet $\mathbf{x}$ is encoded into $B$ coded packets by various coding matrices. The received packet during these $B$ time slots can be represented by

$$\mathbf{y} = \begin{bmatrix} \mathbf{h}(1)^T & \cdots & \mathbf{0}^T \\ \vdots & \ddots & \vdots \\ \mathbf{0}^T & \cdots & \mathbf{h}(B)^T \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{C}(1) \\ \vdots \\ \mathbf{C}(B) \end{bmatrix}}_{\mathbf{C}} \mathbf{x} + \underbrace{\begin{bmatrix} n(1) \\ \vdots \\ n(B) \end{bmatrix}}_{\mathbf{n}}$$

$$= \underbrace{\begin{bmatrix} \mathbf{h}(1)^T\mathbf{C}(1) \\ \vdots \\ \mathbf{h}(B)^T\mathbf{C}(B) \end{bmatrix}}_{\mathbf{H}} \mathbf{x} + \mathbf{n} = \mathbf{H}\mathbf{x} + \mathbf{n}. \qquad (2)$$

With the formulation in (2), the coded end-to-end CRN transmission is identical to a spatial multiplexing MIMO system, except two differences on the equivalent channel matrix $\mathbf{H}$: first, it contains Bernoulli random variables $v_1 \ldots v_R$ other than the path gains in $\mathbf{h}(t)$; secondly, the equivalent channel matrix can be *designed* through the coding matrices $\mathbf{C}(t)$. Consequently, (2) can be regarded as a virtual MIMO system. To keep the throughput of the coded transmission the same as the uncoded transmission, we focus on the design of unity code rate coding matrix, that is, $B$ elements in the data packet $\mathbf{x}$.

The uncoded end-to-end CRN transmission performs poorly owing to the uncertainty of data packet losses. For example, considering a scenario where $R = B = 2$, the uncoded end-to-end CRN transmission can be modeled by using identity matrix as the coding matrix. The equivalent channel matrix is given by

$$\begin{bmatrix} v_1g_1(1) & v_2g_2(1) \\ v_1g_1(2) & v_2g_2(2) \end{bmatrix}, \qquad (3)$$

where the $r$th column in (3) becomes a null vector when the $r$th relay path is occupied by other users ($v_r = 0$). Consequently, the corresponding part of data packet is discarded. In other words, we should design a set of coding matrices $\mathbf{C}(t)$ to avoid such situation.

## III. CODING MATRIX DESIGN

In this section, two coding matrices are introduced based on the permutation matrix and the maximum distance criterion in the coding theory [3]. The rationales include the mapping of the end-to-end CRN transmission into a physical MIMO system and the design of the end-to-end CRN transmission scheme by referring to the MIMO techniques. Fine-grained optimization is not the main focus at this point. In the following paragraphs we assume case with the path number $R$ equal to the elapsed time slots $B$, i.e., $B = R$.

### *A. Permutation Coding Matrix (PCM)*

Owing to the randomness of $v_r$, the equivalent channel matrix of the uncoded end-to-end CRN transmission may contain null column vectors as shown in (3), causing a catastrophic bit error rate (BER) deterioration. Such problem can be greatly mitigated by transmitting the shifted versions of the data packets in different time slots. Specifically, the first coding matrix $\mathbf{C}(1)$ is an identity matrix $\mathbf{I}_R$; other coding matrices $\mathbf{C}(t)$ are generated by circularly shifting the row vectors of $\mathbf{C}(t-1)$ up (or down). A set of permutation matrices is therefore constructed to form the coding matrices. As will be shown in Section IV and Section V, introducing the permutation coding matrix (PCM) effectively lowers the error floor.

### *B. Maximum-Distance Coding Matrix (MCM)*

The PCM significantly improves the communication reliability of the end-to-end CRN transmission by reducing the probability of $\mathbf{H}$ with null columns. Nonetheless, the path diversity is not fully utilized. When $\hat{R}$ paths are occupied, the minimum distance between any two codewords (the total $B$ transmitted coded packets) $\hat{\mathbf{s}} = \mathbf{C}\hat{\mathbf{x}}$ and $\check{\mathbf{s}} = \mathbf{C}\check{\mathbf{x}}$ is $R - \hat{R}$. In this case, the path diversity is reduced to be less than or equal to $R - \hat{R}$ [3]. To combat with this loss, we use an orthonormal matrix with bipolar entries to maximize the minimum distance. Such maximum-distance coding matrix (MCM) can be easily constructed by referring to the full-rate orthogonal space-time code with real entries [9]. For example, the first MCM for a CRN with $R = B = 4$ can be given by

$$\mathbf{C}(1) = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix}. \qquad (4)$$

Other coding matrices are generated in a similar way as mentioned in the previous subsection, that is, circularly shifting the row vectors of $\mathbf{C}(t-1)$ up (or down). In this case, even when only one path is available, the equivalent channel matrix are still constituted by non-zeros entries, leading to the minimum distance being $R$, the same as the case when all paths are available. Finally, it should be emphasized that shifting the row vectors is essential because this preserves the orthogonality of the $r$th row vectors in different MCM. Otherwise, the equivalent channel matrix has the rank deficient problem when some paths are occupied by other users.

## IV. THEORETICAL ERROR RATE ANALYSIS

In this section, the BERs of the uncoded and coded end-to-end CRN transmission are theoretically analyzed. By defining $m = \arg\min_{r,\, 1\le r\le R} P(v_r = 0)$, the occurrence of the CRN with $\hat{R}$ occupied paths in a certain time instance can be lower bounded by

$$\begin{pmatrix} R \\ \hat{R} \end{pmatrix} P(v_m = 0)^{\hat{R}} P(v_m = 1)^{R-\hat{R}}. \tag{5}$$

Then, $\gamma_{ub}(R)$, the BER of the uncoded end-to-end CRN transmissions with $R$ relay paths, can be computed by summing the conditional BERs of all cases with $\hat{R}$ occupied paths and $R - \hat{R}$ available paths, denoted by $\gamma_{ub}(R, \hat{R})$, multiplied by its corresponding occurrence defined in (5)

$$\gamma_{ub}(R) \ge \sum_{\hat{R}=0}^{R} \begin{pmatrix} R \\ \hat{R} \end{pmatrix} P(v_m = 0)^{\hat{R}} P(v_m = 1)^{R-\hat{R}} \gamma_{ub}(R, \hat{R}). \tag{6}$$

At high SNR, $\gamma_{ub}(R, \hat{R})$ is dominated by the data loss through those occupied paths, i.e, $\gamma_{ub}(R, \hat{R}) \ge \big(0 \cdot (R - \hat{R}) + 0.5 \cdot \hat{R}\big)/R = \hat{R}/2R$. Inserting this to (6), we have the theoretical error floor (TEF) of the uncoded end-to-end CRN transmission with

$$\begin{aligned}\gamma_{ub}(R) &\ge \sum_{\hat{R}=0}^{R} \begin{pmatrix} R \\ \hat{R} \end{pmatrix} P(v_m = 0)^{\hat{R}} P(v_m = 1)^{R-\hat{R}} \frac{\hat{R}}{2R} \\ &= \frac{P(v_m = 0)}{2},\end{aligned} \tag{7}$$

where the result is obtained by using change of variables and the Binomial theorem. (7) reveals an interesting fact that the uncoded TEF is independent of the number of relay paths $R$. In other words, increasing the number of relay paths cannot improve the BER of the uncoded end-to-end CRN transmission.

For the coded end-to-end CRN transmission where either the PCM or MCM is adopted, we define $\gamma_{cb}(R, \hat{R})$ as the conditional BER of the case with $\hat{R}$ occupied paths. Since different parts of the packet can be transmitted via different relay paths, $\gamma_{cb}(R, \hat{R})$ at high SNR approaches zero, except that $\gamma_{cb}(R, R) = 0.5$. Consequently, the TEF of the coded end-to-end CRN transmission can be derived as follow

$$\begin{aligned}\gamma_{cb}(R) &\ge \sum_{\hat{R}=0}^{R} \begin{pmatrix} R \\ \hat{R} \end{pmatrix} P(v_m = 0)^{\hat{R}} P(v_m = 1)^{R-\hat{R}} \gamma_{cb}(R, \hat{R}) \\ &\ge \frac{P(v_m = 0)^R}{2}.\end{aligned} \tag{8}$$

Comparing the uncoded TEF (7) and the coded TEF in (8), we observe that the coding matrix leverages the path diversity $R$ to decrease the error floor of the CRN exponentially, thereby significantly enhancing the end-to-end transmission quality in CRN. It is worthwhile mentioning that such closed-form expressions of the error floors clearly show the relations among the BER and system parameters, e.g., the time-out period and the number of relay paths, thereby facilitating the joint design with other techniques such as HARQ in [2].

## V. SIMULATION RESULTS

In this section, the capability of the proposed coding matrices in CRN is demonstrated by means of Monte Carlo simulations. QPSK modulation is adopted and we let $B = R = 8$. The number of relay nodes along a path is assumed to be uniformly distributed between one and three with the i.i.d. Rayleigh link fading gains.

We assume that the receiver has the perfect knowledge of $v_r$. In practice, the receiver can be designed to jointly detect the data packet $\mathbf{x}$ and the randomness $\mathbf{v} = [v_1 \;\ldots\; v_R]^T$ based on the maximum *a posteriori* (MAP) criterion:

$$\begin{aligned}(\hat{\mathbf{x}}, \hat{\mathbf{v}}) &= \arg\max_{\mathbf{x},\mathbf{v}} \log P(\mathbf{x}, \mathbf{v}|\mathbf{y}) \\ &= \arg\max_{\mathbf{x},\mathbf{v}} -||\mathbf{y} - \mathbf{H}\mathbf{x}||^2 + \sum_{r=1}^{R} \log P(v_r).\end{aligned} \tag{9}$$

Compared to the conventional MIMO detection in which $\mathbf{x}$ is the only unknown variable, jointly detecting $(\mathbf{x}, \mathbf{v})$ demands at most $2^R$ sets of the conventional MIMO detectors for each possible value of $\mathbf{v}$, that is, the exhaustive search (ES) of $\mathbf{v}$. However, the sphere decoder (SD) [10] can be applied to the joint detection here. By converting the joint detection into a constrained tree search where many elements in the search space are pruned during search, the SD is proven to be an efficient algorithm that significantly reduces the detection complexity compared with ES. Thus, the complexity of using $2^R$ sets of MIMO detectors is a loose upper bound and the actual complexity should be much less than that.

For both uncoded and coded end-to-end CRN transmissions with eight relay paths ($R = 8$), Fig. 2 depicts the numerical results of conditional BERs $\gamma_{ub}(R, \hat{R})$ and $\gamma_{cb}(R, \hat{R})$. Filled markers are used to highlight the proposed PCM and MCM. As can be seen, the coding matrix removes the error floor since as long as $\hat{R} < R$, all the entries in $\mathbf{x}$ can be transmitted from available paths. The CRN end-to-end transmission with MCM has the steepest BER curves, reflecting its highest path diversity. We also simulate the repetition-based scheme (RBS) which transmits duplicate data symbol for all relay paths. Such RBS can be regarded as a coded transmission where the coding matrix contains one column with all entries being one and the rest columns being null vectors. We can see that the RBS is outperformed by the proposed PCM and MCM. In the next two simulations, as same transition probabilities $(p_{01,r}, p_{10,r})$ for all relay paths are used, the index $r$ of the transition probabilities is dropped. For $(p_{10}, p_{01}) = (0.5, 0.5)$, Fig. 3 shows the numerical BERs and the TEFs under various $W$. The larger $W$ implies that the destination nodes allow longer time-out period, resulting in the smaller $P(v_r = 0)$. Again, the uncoded end-to-end CRN transmission suffers from the poor error rate performance, and coding matrix effectively reduces

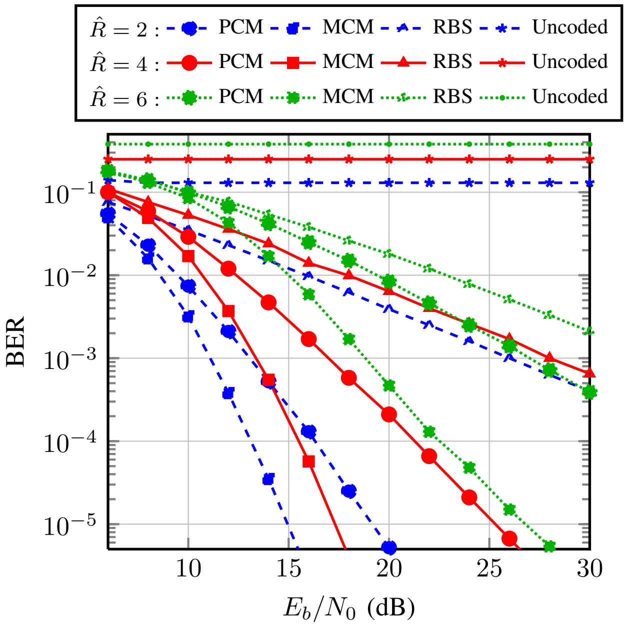

Fig. 2. Conditional BER for various number of unavailable paths $\hat{R}$ in uncoded and coded (PCM, MCM, and RBS) end-to-end CRN transmissions; $B = R = 8$.

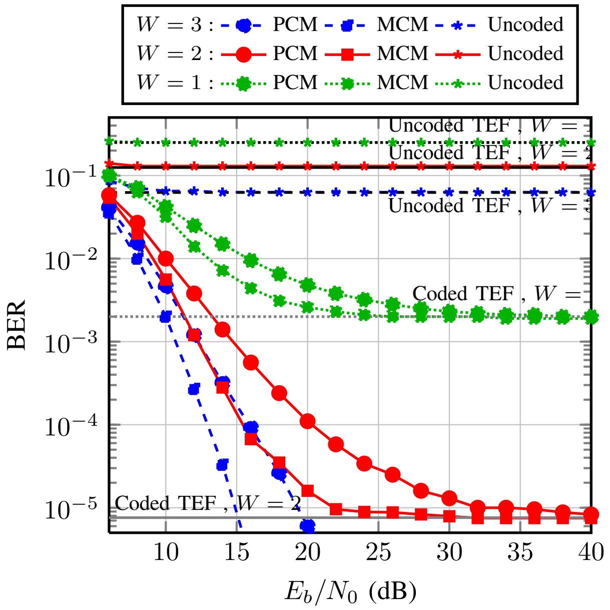

Fig. 3. BER for various time-out period in uncoded and coded (PCM and MCM) end-to-end CRN transmissions; $(p_{01}, p_{10}) = (0.5, 0.5)$, $B = R = 8$. For $W = 3$, the coded TEF in (8) is $3 \times 10^{-8}$.

the error floors. Comparing between the two proposed coding schemes, the MCM delivers the BER performance superior to the PCM does. Last, we can see that both the uncoded TEF in (7) and the coded TEF in (8) are tightly matched with the numerical results, validating our theoretical analyses. Other cases of $(p_{10}, p_{01})$ also show the similar behaviors.

Using the coded TEF expression, Fig. 4 shows the number of required relay paths of coded CRNs with various time-out period $W$ of receiving packets in order to achieve an acceptable BER of $10^{-3}$. As $W$ or $p_{01}$ increases, less relay paths are required to achieve the target BER, owing to the exploitation of path diversity. For low $p_{01}$, longer time-out time is more efficient as the number of required relay paths $R$ can be significantly reduced, whereas at high $p_{01}$, adding a few more relay paths greatly shorten the time-out time and so does the latency. The clear TEF expression in (8) helps us to design and analyze the CRN.

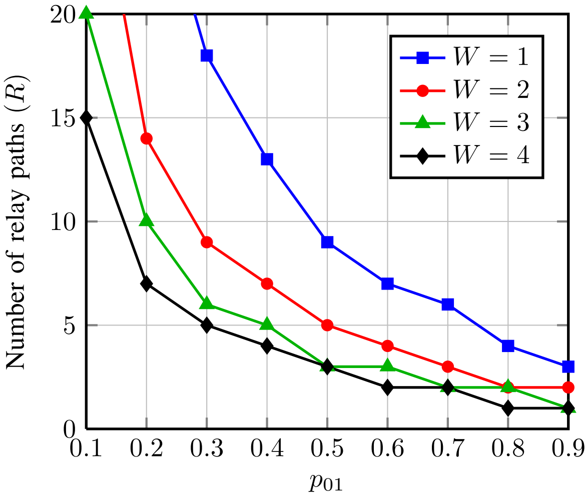

Fig. 4. Theoretical required number of relay paths $R$ vs. various state transition probabilities for coded end-to-end CRN transmissions; the target BER is $10^{-3}$.

## VI. CONCLUSION AND DISCUSSION

This letter shows the feasibility of the coded end-to-end transmission in CRN to reduce error floor and to shorten latency by controlling the number of relay paths. We treat the coding matrix as the *path-time code (PTC)*, to open the door for the *virtual MIMO* research for CRN or general ad hoc networks with fading and interference.

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